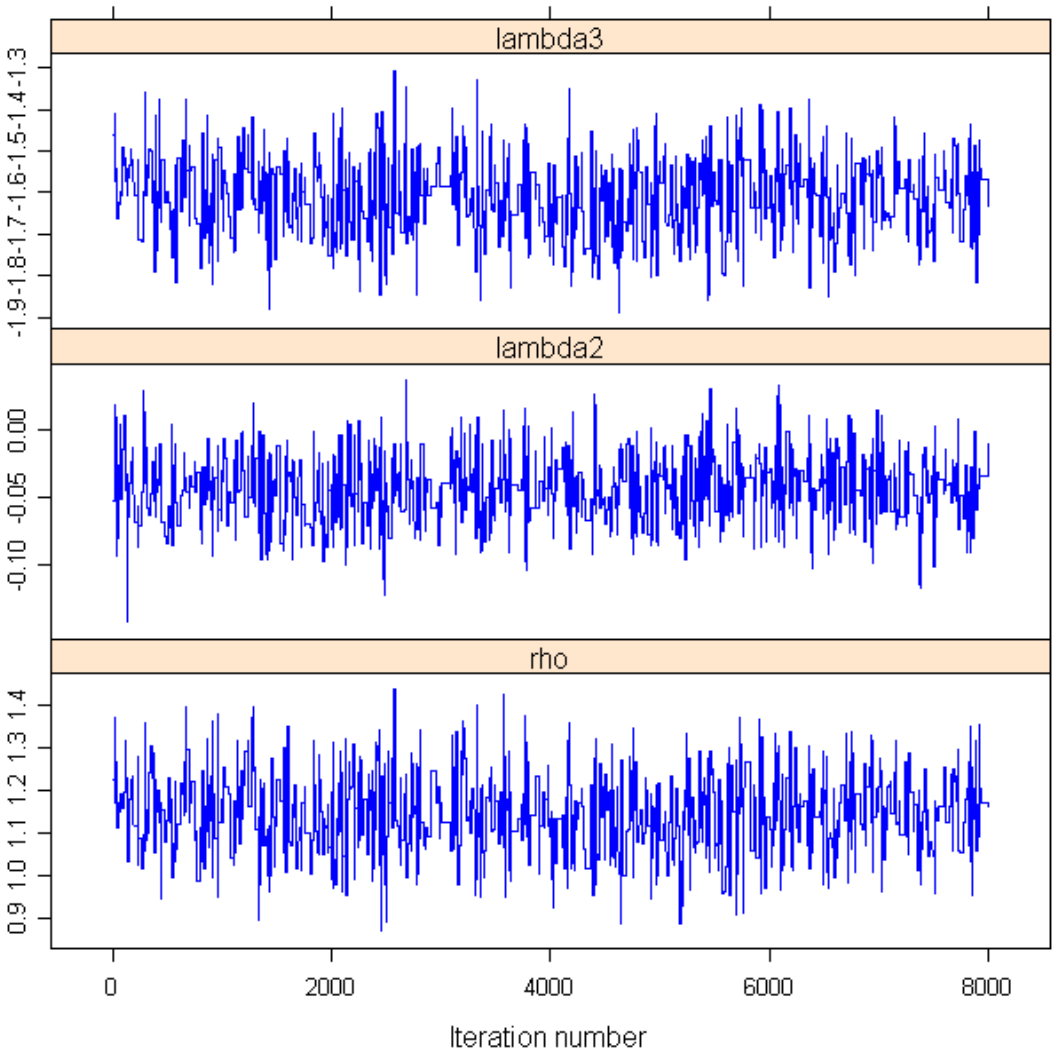
## Standard Maximum Likelihood

```
results
            [,1]       [,2]       [,3]         [,4]
[1,]  1.14217375 0.08810331  12.964027 1.187315e-06
[2,] -0.04587087 0.02420965  -1.894735 9.473289e-02
[3,] -1.60468315 0.09017014 -17.796169 1.017642e-07
```

## rwmetrop: Random Walk Metropolis from LearnBayes Package



```
          Summary of the Chains Using Coda Package
mysummary

Iterations = 1:8000
Thinning interval = 1
Number of chains = 1
Sample size per chain = 8000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

            Mean      SD  Naive SE Time-series SE
rho      1.14282 0.08472 0.0009472       0.004605
lambda2 -0.04426 0.02300 0.0002571       0.001149
lambda3 -1.60847 0.08631 0.0009650       0.004791

2. Quantiles for each variable:

            2.5%      25%      50%      75%     97.5%
rho      0.97757  1.08838  1.13852  1.19964  1.309601
lambda2 -0.08867 -0.05986 -0.04402 -0.03043  0.001701
lambda3 -1.78011 -1.66818 -1.60579 -1.54622 -1.444427
```

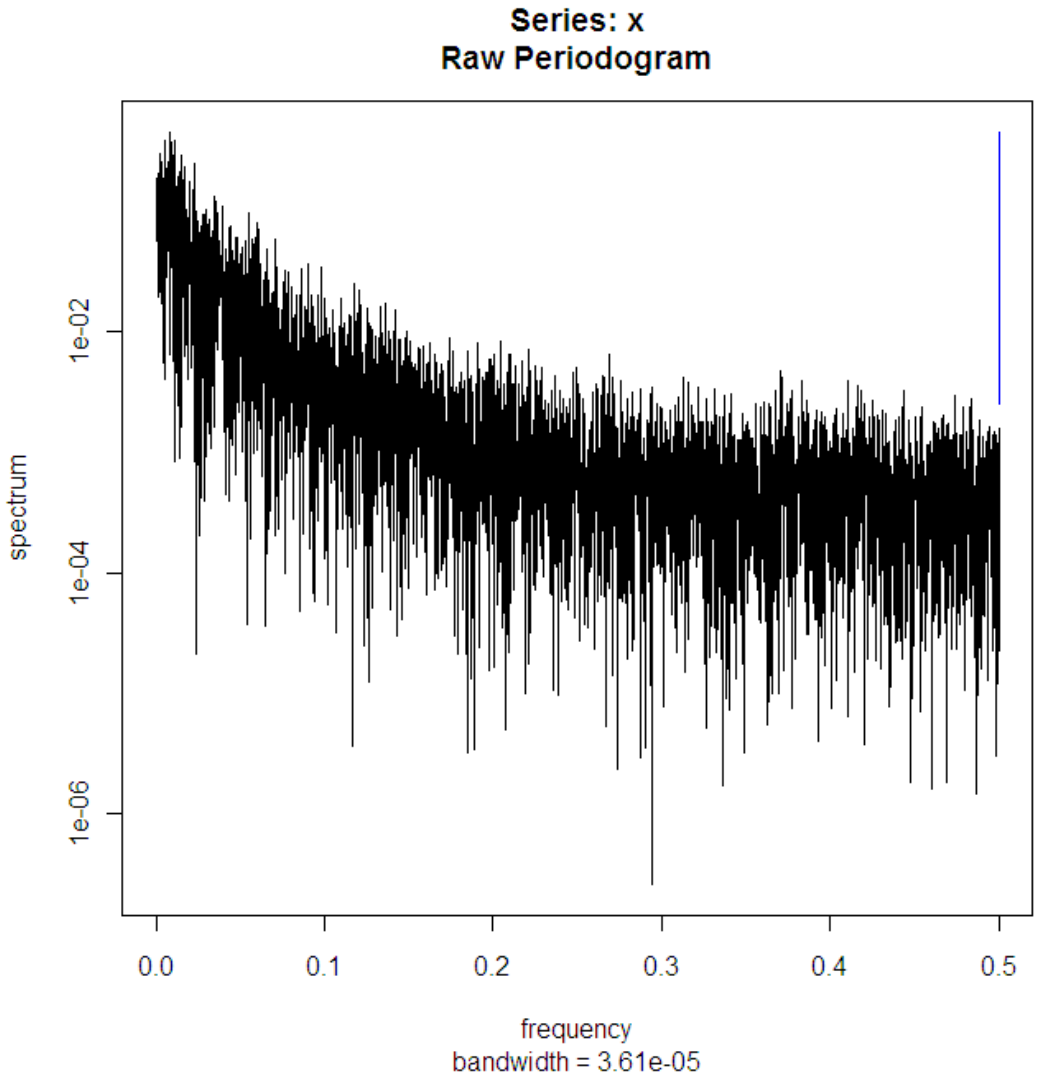
Here are the formulas for the series statistics:

```
results2[j,1] <- mean(kingmetrop$par[-c(1:2000),j])
results2[j,2] <- sd(kingmetrop$par[-c(1:2000),j])
results2[j,3] <- sd(kingmetrop$par[-c(1:2000),j])/sqrt(length(kingmetrop$par[-
c(1:2000),j]))
xheck <- spectrum0(kingmetrop$par[-c(1:2000),j], max.freq = 0.5, order = 1, max.length =
200)
results2[j,4] <- sqrt(xheck$spec/length(kingmetrop$par[-c(1:2000),j]))
> results2
           [,1]       [,2]         [,3]        [,4]
[1,]  1.14281978 0.08472390 0.0009472420 0.004604928
[2,] -0.04425939 0.02299666 0.0002571105 0.001149165
[3,] -1.60847050 0.08630855 0.0009649590 0.004791468
```

The “Time Series SE” is equivalent to batching of the data (see Albert for a discussion). The idea is to compute means for chunks of the time series (after burnin) and then compute the standard error of those means. Later a fancier method that essentially is equivalent to analyzing the *spectral density* of the time series (see below).

**Spectral Analysis of time series for Lamba3**

spectrum((mcmc(kingmetrop$par[-c(1:2000),3])))

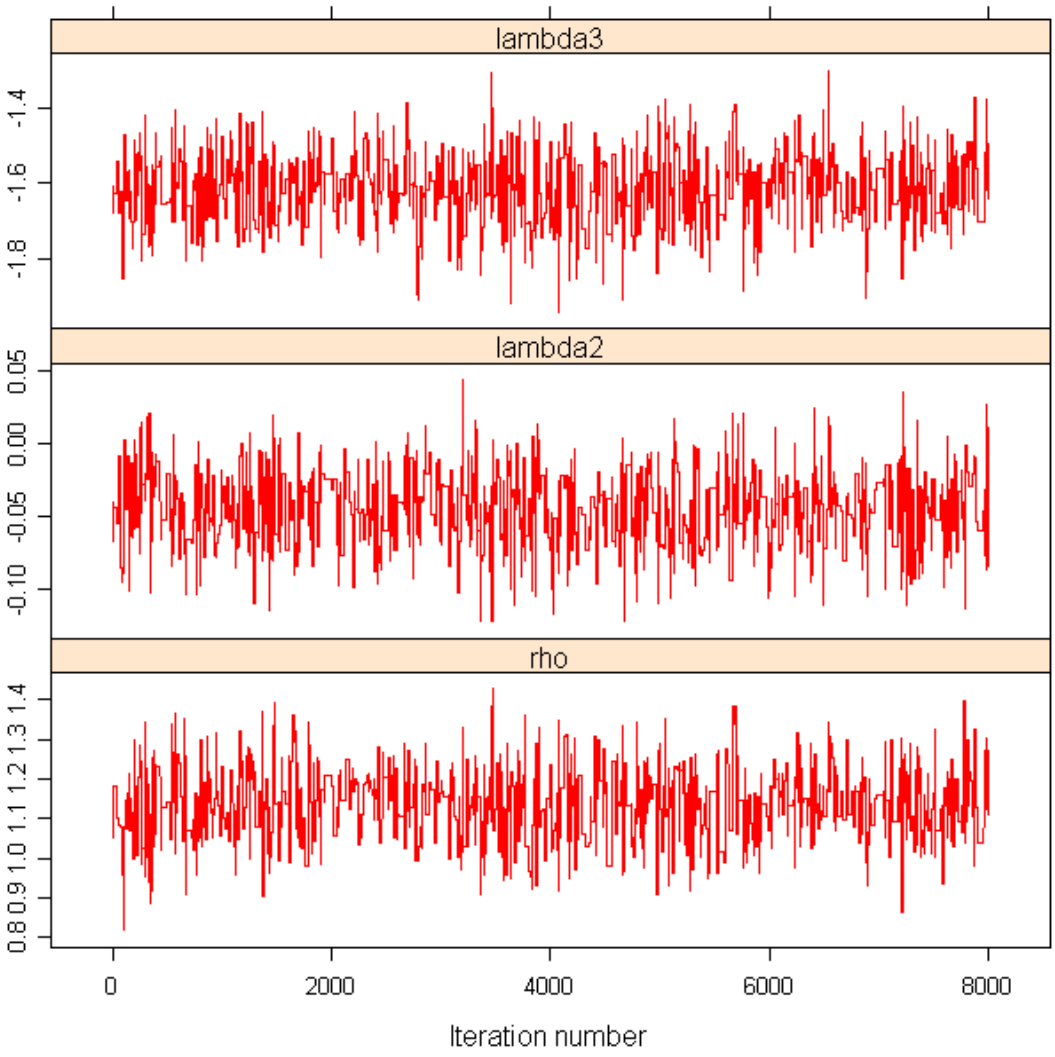



The spectral density at frequency zero is estimated by fitting a glm to the low-frequency end of the periodogram. `spectrum0(x)/length(x)` estimates the variance of `mean(x)`.

```
xheck <- spectrum0(kingmetrop$par[-c(1:2000),1], max.freq = 0.5, order
= 1, max.length = 200)
xheck$spec = 0.1696429 -- This is related to a transformation of the
variance of the first 200 points plotted above
sqrt(xheck$spec/length(kingmetrop$par[-c(1:2000),1])) = 0.004604928
>
> results22 -- Time-series SE Using 40 Group Means of Size 200 each
            [,1]        [,2]        [,3]
[1,] 0.003815361 0.001155490 0.003811359
```

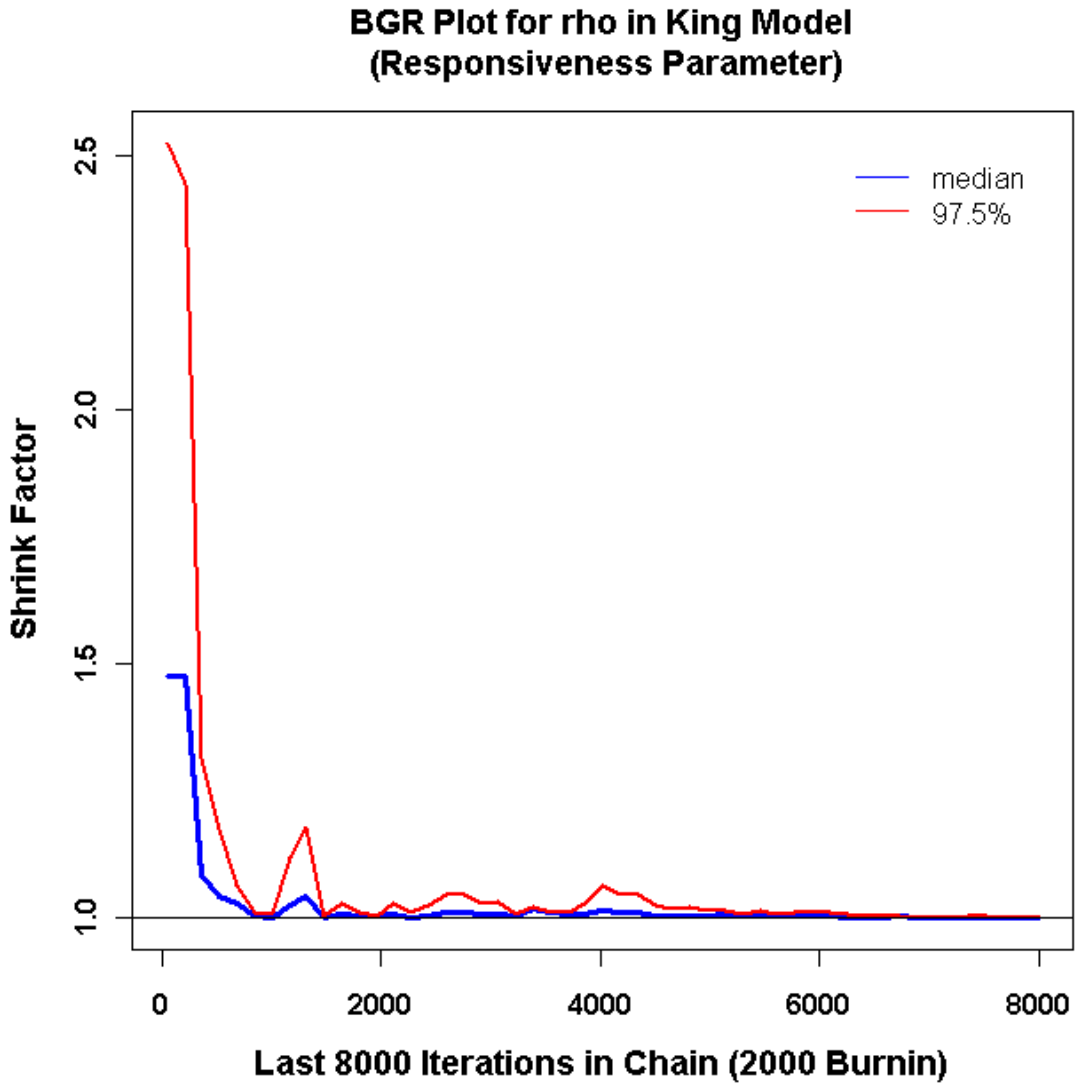
>

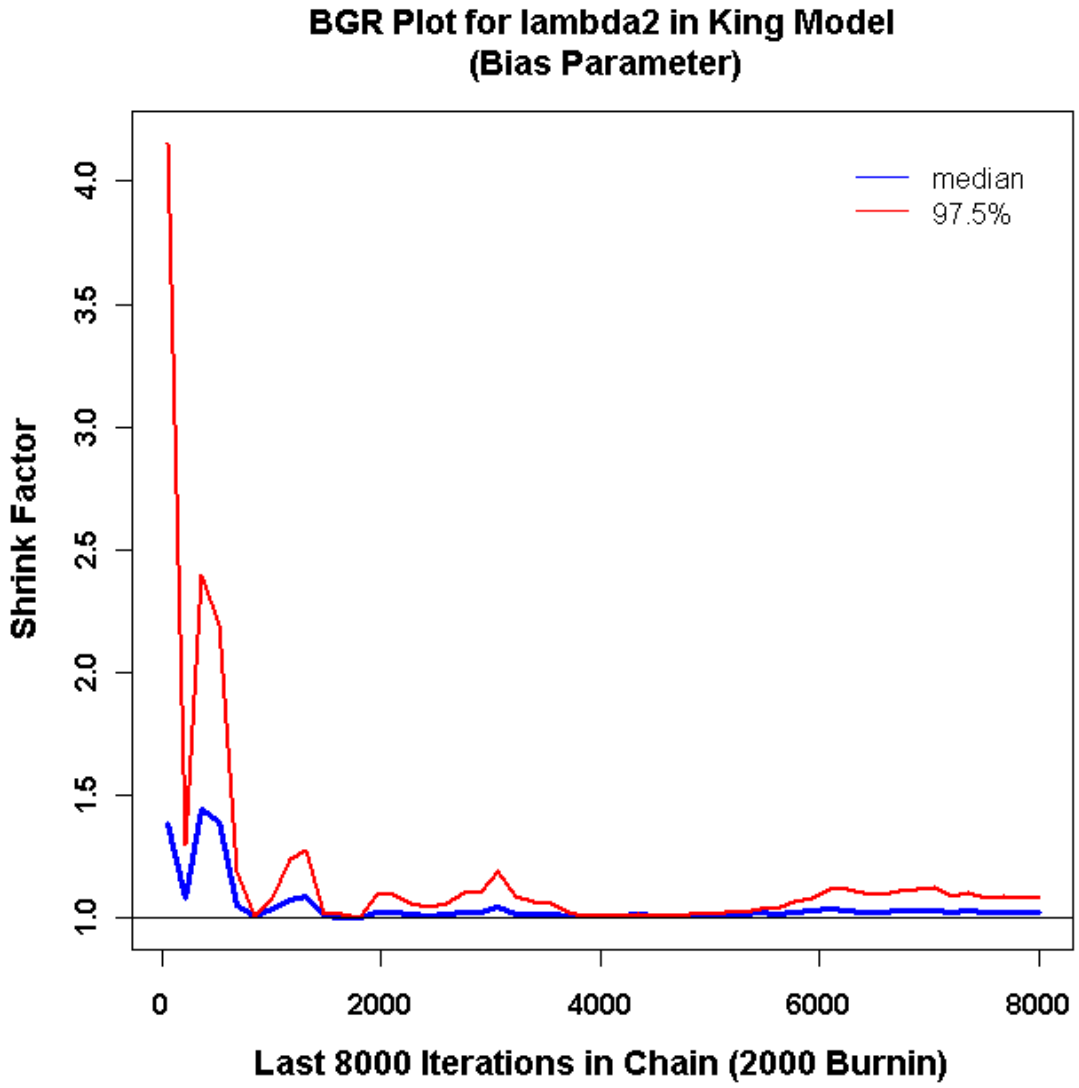
**rwmetrop: Random Walk Metropolis from LearnBayes Package with very diffuse starts**

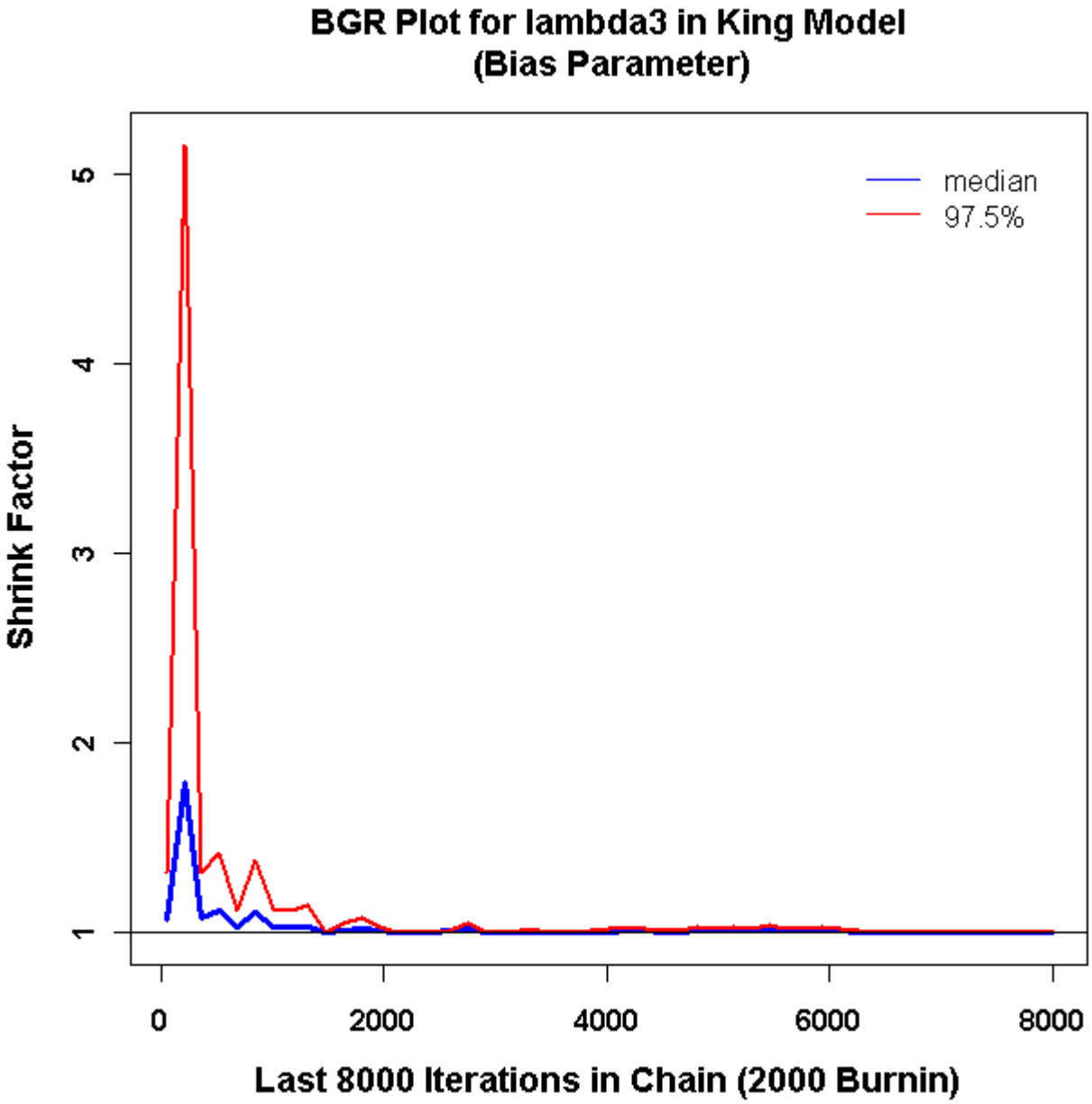


## Summary of the Chains Using Coda Package

```
 mysummary2

Iterations = 1:8000
Thinning interval = 1
Number of chains = 1
Sample size per chain = 8000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

            Mean      SD  Naive SE Time-series SE
rho      1.13467 0.08727 0.0009758       0.004578
lambda2 -0.04534 0.02484 0.0002777       0.001074
lambda3 -1.61632 0.09007 0.0010071       0.004630

2. Quantiles for each variable:

            2.5%      25%      50%      75%      97.5%
rho      0.96231  1.07759  1.13677  1.19286  1.3050367
lambda2 -0.09666 -0.06288 -0.04413 -0.02723 -0.0005541
lambda3 -1.79294 -1.67666 -1.61709 -1.55382 -1.4386562

results3 – directly calculated as a check
            [,1]       [,2]         [,3]        [,4]
[1,]  1.13466738 0.08727419 0.0009757551 0.004578457
[2,] -0.04533884 0.02483643 0.0002776797 0.001074399
[3,] -1.61632489 0.09007436 0.0010070620 0.004630268

results33 -- Time-series SE Using 40 Group Means of Size 200 each
            [,1]        [,2]        [,3]
[1,] 0.004057944 0.001024239 0.004417255
```


BGR Plot for rho in King Model
(Responsiveness Parameter)
median
97.5%
Shrink Factor
1.0
1.5
2.0
2.5
0
2000
4000
6000
8000
Last 8000 Iterations in Chain (2000 Burnin)


BGR Plot for lambda2 in King Model
(Bias Parameter)
median
97.5%
Shrink Factor
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0
2000
4000
6000
8000
Last 8000 Iterations in Chain (2000 Burnin)




**The Gelman-Rubin Statistics from gelman.diag function – These are the final values above**

```
 results4
     Point est. 97.5% quantile
         [,1]     [,2]
[1,] 1.000486 1.002733
[2,] 1.022378 1.090191
[3,] 1.001450 1.001695
```

## Autocorrelation Plots of the two Random Walk Metropolis Chains