

Turnout: logit/probit models for binary data

Voter turnout is coded as a binary indicator ($y = 1$ if voted, 0 otherwise) and related to covariates via a logit model. The data comprise a random 3,000 observation subset of a much larger data set analyzed by Jonathan Nagler.

Logit model:

A logit model for binary data is easily implemented in WinBUGS, with the `dbern()` sampling density and the `logit()` link function:

```

model{
  for (i in 1:N){                                ## loop over observations
    y[i] ~ dbern(p[i]);                         ## binary outcome, Bernoulli trial
    logit(p[i]) <- mu[i];                      ## logit link
    mu[i] <- beta[1]                            ## regression structure for covariates
    + beta[2]*educ[i]
    + beta[3]*(educ[i]*educ[i])
    + beta[4]*age[i]
    + beta[5]*(age[i]*age[i])
    + beta[6]*south[i]
    + beta[7]*govelec[i]
    + beta[8]*closing[i]
    + beta[9]*(closing[i]*educ[i])
    + beta[10]*(educ[i]*educ[i]*closing[i]);
  }

  ## extra quantity, not necc for model fitting
  llh[i] <- y[i]*log(p[i]) + (1-y[i])*log(1-p[i]); ## llh contribution
}

sumllh <- sum(llh[]);      ## sum of log-likelihood contributions

## priors, hyper-parameters b0 and B given at end of data file
beta[1:10] ~ dmnorm(b0[ ] , B[ , ]); ## multivariate Normal prior
}

```

Data:

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 5, 4, 4, 6, 5, 5, 1, 5, 4, 2, 6, 6, 6, 5, 5, 5, 5, 3, 5, 5, 4, 2, 8,
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Initial Values:

```
## reasonable start values
list(beta = c(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0))

## alternative set of start values for 2nd parallel chain, from OLS
list(beta=c(-0.3392423378, 0.0741911953, 0.0012163747, 0.0230970246,
-0.0001679677, -0.0333484965, 0.0204799754, -0.0068319918,
0.0017752978, -0.0001432201))
```

Node	mean	sd	MC error	2.5%	median	97.5%	start	sample
beta[1]	-3.382	0.8583	0.0831	-5.058	-3.21	-2.016	1	10000
beta[2]	-0.08477	0.3112	0.03091	-0.4632	-0.2237	0.5335	1	10000
beta[3]	0.06473	0.03231	0.003187	-2.651E-4	0.07804	0.1077	1	10000
beta[4]	0.1076	0.01625	0.001535	0.06975	0.1075	0.1351	1	10000
beta[5]	-7.446E-4	1.65E-4	1.537E-5	-0.001028	-7.443E-4	-3.512E-4	1	10000
beta[6]	-0.1639	0.1029	0.001773	-0.3656	-0.1637	0.03958	1	10000
beta[7]	0.1121	0.1131	0.00174	-0.1101	0.1125	0.3336	1	10000
beta[8]	-0.04739	0.03025	0.002957	-0.09905	-0.05105	0.01363	1	10000
beta[9]	0.01779	0.01223	0.001217	-0.006045	0.02089	0.03773	1	10000
beta[10]	-0.001895	0.001262	1.246E-4	-0.003868	-0.002257	4.841E-4	1	10000

FROM MANUAL: MC Error: Similar to the standard error of the mean but adjusted for autocorrelated sample. It will get smaller as more iterations are run. One way to assess the accuracy of the posterior estimates is by calculating the Monte Carlo error for each parameter. This is an estimate of the difference between the mean of the sampled values (which we are using as our estimate of the posterior mean for each parameter) and the true posterior mean.

As a rule of thumb, the simulation should be run until the Monte Carlo error for each parameter of interest is less than about 5% of the sample standard deviation. The Monte Carlo error (MC error) and sample standard deviation (SD) are reported in the summary statistics table.



