The One Parameter logistic model (the Rasch model)

Let θ_j, j=1,,,q, be the jth subject's level of ability.

Let δ_i , i=1,,,p, be the ith item -- the level of difficulty of the ith question.

Then the probability that subject i gets test question j correct is:

$$P(z_{ij}=1|\,\theta_j,\delta_i)=\frac{e^{\alpha(\theta_j-\delta_i)}}{1+e^{\alpha(\theta_j-\delta_i)}}$$

where z_{ij} is an indicator variable that is equal to 1 if subject j answers question i correctly, and 0 otherwise; and α determines the shape of the item response function. If $\alpha\to\infty$ then $P(z_{ij}=1)$ if $\theta_j > \delta_i$. That is, it collapses to a Guttman Scale.

Assuming independence of the responses; that is:

$$P\left(z_{ij}=1\cap z_{\ell j}=0\right)=P\left(z_{ij}=1\right)P\left(z_{\ell j}=0\right)$$

Then

$$\frac{P\left(z_{ij}=1\cap z_{\ell j}=0\right)}{P\left(z_{ij}=0\cap z_{\ell j}=1\right)}=\frac{P\left(z_{ij}=1\right)P\left(z_{\ell j}=0\right)}{P\left(z_{ij}=0\right)P\left(z_{\ell j}=1\right)}=$$

$$\frac{\dfrac{e^{\alpha(\theta_j-\delta_i)}}{1+e^{\alpha(\theta_j-\delta_i)}}*\dfrac{1}{1+e^{\alpha(\theta_j-\delta_\ell)}}}{\dfrac{1}{1+e^{\alpha(\theta_j-\delta_i)}}*\dfrac{e^{\alpha(\theta_j-\delta_\ell)}}{1+e^{\alpha(\theta_j-\delta_\ell)}}}=\frac{e^{\alpha(\theta_j-\delta_i)}}{e^{\alpha(\theta_j-\delta_\ell)}}=e^{\alpha(\theta_j-\delta_i-\theta_j+\delta_\ell)}=e^{\alpha(\delta_\ell-\delta_i)}$$

So the individual parameter cancels leaving just the difference between the two question parameters.