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Directional and Euclidean Theories Of Voting Behavior: A Legislative Comparison

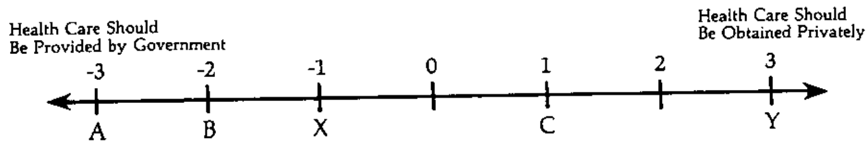
Recent research by Rabinowitz and Macdonald (1989) claims that voting behavior is better accounted for by a directional model than by a traditional proximity or Euclidean model. This paper compares directional and Euclidean models using congressional roll-call voting data. For these relatively well-informed voters, we can unambiguously reject the directional model in favor of the traditional Euclidean spatial model. We conclude that congressional voting can indeed be very accurately represented by the Euclidean model.

Recent research by Rabinowitz and Macdonald (1989) claims that voting behavior is better accounted for by a directional model than by a traditional proximity or Euclidean model. This paper presents an empirical examination of their model using congressional roll-call voting data. We argue that the conclusions reached by Rabinowitz and Macdonald do not generalize to a body of well-informed voters. We find that when their model is tested with congressional data, we can unambiguously reject the directional model of voting behavior.

The Euclidean model asserts that, in a spatial representation, utility is monotonically decreasing in distance from the ideal point. Since the seminal multidimensional work of Davis, Hinich, and Ordeshook (1970), the Euclidean model has become the standard for formal theoretical and empirical work in political science.[1]

The directional model is based upon a formal theory of voting in which voters have diffuse preferences over alternatives. Drawing upon cognitive psychology, the directional model is developed by arguing that voting decisions are made in response to symbolic stimuli. Voters, first and foremost, determine on which side of an issue the candidate lies (a binary decision) and then determine how far from neu-

FIGURE 1
Health Care Issue



tral or how "intense" both they and the candidate are. This effect can be expressed in the following utility function of a voter with a vector of ideal points, $\mathbf{X}_i$, over a candidate with a vector of ideal points, $\mathbf{Z}_j$

$$U(\mathbf{Z}_j) = (\mathbf{Z}_j - \mathbf{X}_0)'(\mathbf{X}_i - \mathbf{X}_o) \tag{1}$$

where $\mathbf{X}_0$ is the neutral point. Following the example used in Rabinowitz and Macdonald, Figure 1 shows the one-dimensional interpretation of this preference relation.

In Figure 1, X and Y are voters and the candidates are A, B, and C. Directional theory implies that X prefers A to either B or C and prefers B to C. It also implies that Y prefers C to either B or A and prefers B to A. Note that proximity theory (according to which voters choose the candidate closest in Euclidean distance) would make the same predictions for voter Y but very different predictions for voter X. For instance, proximity theory would tell us that, given a choice between candidates A and B, voter X would rather vote for B, since B's ideal amount of public health care is more similar, or closer, to X's ideal than is A's. But directional theory implies that X will vote for the candidate that locates farthest left on the axis.[2]

In order to test their hypothesis, Rabinowitz and Macdonald use data and thermometer scores from the National Election Studies in which voters place themselves on 7-point issue scales; each presidential candidate is placed on these scales at the mean of voters' perceptions of the candidate's position. They note that squared Euclidean distance between the ideal point of candidate **Z** and the ideal point of voter **X** in multidimensional space can be represented by

$$|\mathbf{Z}|^2 + |\mathbf{X}|^2 - 2\,|\mathbf{Z}|\,|\mathbf{X}| \cos \mathbf{Z}'\mathbf{X} \tag{2}$$

They estimate the following regression equation to obtain estimates for the regression coefficients:

$$CE = \beta_0 + \beta_1 [- |\mathbf{Z}|^2 - |\mathbf{X}|^2] + \beta_2 [2|\mathbf{Z}| |\mathbf{X}| \cos \mathbf{Z'X}] + \text{controls} + \varepsilon \quad (3)$$

where CE is candidate evaluations.

This model, as Rabinowitz and Macdonald note, nests both the quadratic model, the most common Euclidean form in empirical and theoretical work, and the directional model.[3] When $\beta_1=\beta_2 >0$; equation (3) collapses to the quadratic model. When $\beta_1=0$ and $\beta_2>0$, equation (3) collapses to the directional model. In their empirical work, Rabinowitz and Macdonald estimate the β's by simple regression and find $\beta_2>>\beta_1$, providing extremely strong support for the directional model.

This strong support for the directional model may reflect levels of voter information. As Rabinowitz and Macdonald recognize (1989, 94), a large proportion of the voting public is uninformed (Campbell, Converse, Miller, and Stokes 1964). Palfrey and Poole (1987) find that there is a very strong central tendency of voter ideal points when the population is uninformed. This central tendency can clearly influence estimation of equation (3) from survey data. Recently, Husted, Kenny, and Morton (1990) used the NES data to make the empirical argument that uninformed voters are more likely to incorrectly assess their representative's ideal point (that voters make such an assessment correctly is one of the key assumptions of the Euclidean model). They show that, as voters acquire more information, they update (in a Bayesian sense) to a point closer to their candidates' actual ideal point. We argue that the results obtained by Rabinowitz and Macdonald imply that the directional theory of voting has implications about how uninformed voters make voting decisions. More importantly, the uninformed voter is more likely to recognize direction only (for example, for or against legalized abortion) and not the more subtle issue of distance (for example, how much federal funding should be designated for women's health services). So it is not surprising that there is strong evidence to support the directional theory when tested on uninformed voters. But the Rabinowitz and Macdonald study does not address the question of whether the theory can be generalized to include informed voters.

The Palfrey and Poole (1987) results imply that we should expect the ideal points of an informed group of voters to be bimodally distributed over the range of ideal points. We should expect the converse to be true of a set of uninformed voters—that is, the variance in ideal points of a sample of uninformed voters should be rather low because their ideal points will cluster in the center.[4]

On the NES issue scales, uninformed respondents tend to place themselves at 4—which may actually mean "don't know"—and this tendency suggests why Rabinowitz and Macdonald found strong support for the directional model. Since the 7-point NES scales range from 1 to 7, Rabinowitz and Macdonald take 4 to be the neutral point. If 4 is subtracted from all scale values, 0 becomes the neutral point and the utility measure used by Rabinowitz and Macdonald can be rewritten as:

$$u_{ij} = -\sum_{k=1}^{s} (z_{jk}-x_{ik})^2 = -\beta_1 \left[\sum_{k=1}^{s} z_{jk}^2 + \sum_{k=1}^{s} x_{ik}^2\right] + \beta_2 \left[2\sum_{k=1}^{s} z_{jk}x_{ik}\right] \quad (4)$$

$$= -\beta_1 \sum_{0} z_j^2 - \beta_1 \left[\sum_{\varnothing} z_j^2 + \sum_{\varnothing} x_i^2\right] + \beta_2 \left[2\sum_{\varnothing} z_j x_i\right] \quad (5)$$

where z_j denotes candidate j's position,
x_i denotes individual i's position,
s is the number of NES issue scales used,
k indexes the scales,
0 is the set of issue scales for which the respondent gave a response of 0 (4 in the actual data), and
$\varnothing$ indicates the nonzero scales.

If a respondent gives a 0 self-placement on all issue scales, the purely directional model (β_1=0) predicts that all candidates will have equal utility for the voter, since the cross-product terms vanish. This is the correct prediction for totally uninformed voters; in contrast, a quadratic model will have these voters influenced by the z_j^2 terms, where the z_j represent average placements made by both uninformed and informed voters. If we estimated the Rabinowitz and Macdonald model on voters with only 0 self-placements, we might well expect to find an estimate for β_1 close to 0. In the full sample, the estimate of β_1 will be influenced both by zero responders and by nonzero responders, whereas the estimate of β_2 will be influenced only by the more informed nonzero responders. Consequently, it is not surprising that $\beta_2 > \beta_1$ when the empirical sample contains a mixture of informed and uninformed types. When the empirical sample consists of highly informed individuals, such as members of Congress, we should expect to find $\beta_1 \simeq \beta_2$.

In this paper we test the Macdonald and Rabinowitz model using congressional roll-call data. We thus move to a legislative setting which is, we believe, relatively free from informational distortions.

The Model

Using data from the House of Representatives for the congressional sessions that overlapped the years Rabinowitz and Macdonald analyzed (the 92d through the 99th Congresses)[5], we estimate a limited dependent variable version of equation (3). Let p denote the number of legislators (the voters in this model), i = 1,. . ., p; let q denote the number of roll-call votes, j = 1, . . . , q; and let s denote the number of policy dimensions, k = 1, . . . , s. Let $\mathbf{x}_i$ be a vector of length s which is the ith legislator's ideal point in the policy space. Each roll-call vote is represented by two policy outcomes, $\mathbf{z}_{jy}$ and $\mathbf{z}_{jn}$, where the y and the n stand for the policy outcomes associated with the yea and the nay votes, respectively.

Legislator i's utility for outcome y on roll call j is

$$U_{ijy} = u_{ijy} + \varepsilon_{ijy} \tag{6}$$

and

$$u_{ijy} = -\beta_1 \left[\sum_{k=1}^{s} \left((x_{ik}-x_o)^2 + (z_{jyk}+x_0)^2 \right) \right] + \beta_2 \left[2\sum_{k=1}^{s} (x_{ik}-x_o)(z_{jyk}-x_o) \right] \tag{7}$$

where u_{ijy} is the deterministic portion of the utility function and ε_{ijy} is the idiosyncratic portion of the utility function. Because we have data on dichotomous choices rather than utilities, it is appropriate to use a limited dependent variable model rather than regression. We refer to (6) and (7) as the full model. It is well known (Dhrymes 1978) that estimation of such models reflects only utility differences rather than utility values. It is also the case, since all **z**'s and **x**'s will be estimated, that we can, without loss of generality, set $\mathbf{x}_0 = 0$. (Doing so would correspond to Rabinowitz and Macdonald's transforming the 7-point scales to make 4 the neutral point.) We accomplish this by estimating $\mathbf{x}_0$ and subtracting it from the **z**'s and **x**'s, thus renormalizing the equation to have a neutral point of zero. In order to avoid estimation problems (see

Poole and Rosenthal 1985, 1991), we will consider the two roll-call outcomes as functions of their midpoint, $\mathbf{z}_{jm}$, and of the distance, $2\mathbf{d}_j$, between them; namely the yes outcome $\mathbf{z}_{jy} = \mathbf{z}_{jm} - \mathbf{d}_j$ and the no outcome $\mathbf{z}_{jn} = \mathbf{z}_{jm} + \mathbf{d}_j$.

We will assume that the stochastic term, ε, is distributed as the log of the inverse exponential (the logit distribution). The probability that legislator i votes for outcome y on roll call j can then be written as

$$\text{Prob(Yea)} = P_{ijy} = \frac{\exp[u_{ijy}]}{\exp[u_{ijy}] + \exp[u_{ijn}]} = \frac{1}{1 + \exp[u_{ijn} - u_{ijy}]} \tag{8}$$

and the likelihood function as

$$L = \prod_{i=1}^{p} \prod_{j=1}^{q} \prod_{\ell=1}^{2} P_{ij\ell}^{C_{ij\ell}} \tag{9}$$

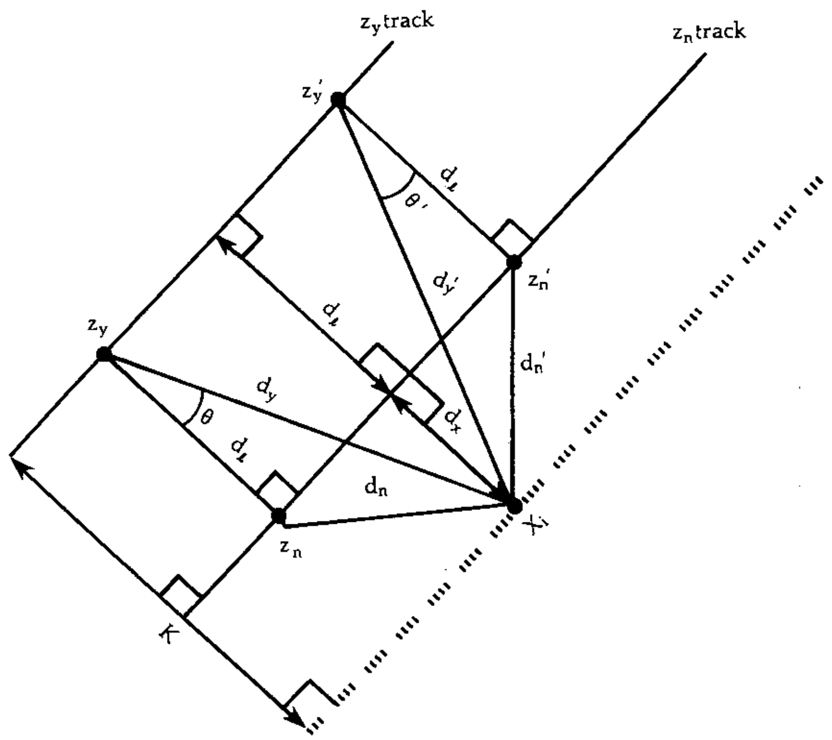
where ℓ is an index for y or n and $c_{ij\ell}$ takes on the value of 1 if choice ℓ is made and 0 otherwise. We estimate the model using a version of the D-NOMINATE algorithm described in detail in Poole and Rosenthal (1991).

Identification Problems

It is straightforward to show that all parameters in (7) are identified in a one-dimensional (s = 1) model if $\beta_1 \neq 0$. However, as inspection of (8) suggests, the **z**'s are not identified in the case of directional voting ($\beta_1 = 0$); only the difference $\mathbf{z}_y - \mathbf{z}_n$ is identified. If the directional model held, the D-NOMINATE algorithm would blow up rather than converge.

Another identification problem is encountered when estimating the multidimensional model. Consider the case of two dimensions [let s = 2 in equation (7)]. If we actually observed utility differences and had nonzero values for the β's and values for the **x**'s, we could regress the differences on a constant and the x_{i1} and x_{i2} values for each roll call. This would give us only three coefficients with which to estimate the four z's for the roll call. So the **z**'s are not identified. The problem is illustrated graphically in Figure 2. The Pythagorean theorem shows that the difference in squared distances from the **z**'s to the **x**'s is constant as the **z**'s move along the track. The observed data would allow us to identify the distance between the alternatives, the distance of the track from **x**, the angle between **x** and a perpendicular to the track, but not the locations of the alternatives along the track. For the directional model, things are even worse, since the constant

FIGURE 2
Example of Identification Problem



Note: The logit procedure estimates utility differences, $\Delta U = \beta[\, d_y^2 - d_n^2 \,]$. Figure 2 shows that both (z_y, z_n) and (z_y', z_n') or *any* z_y's or z_n's along the tracks will give equal values for all ΔU. Consider the triangle (z_y, z_n, X_i), defined by

$$d_n^2 - d_y^2 = d_\ell^2 - 2 d_y d_\ell \cos\theta$$

Since $\cos\theta d_y = K = \cos\theta' d_y'$, $d_n^2 - d_y^2 = d_\ell^2 - 2 d_\ell K$, which is invariant for z_y and z_n on the tracks.

would drop out of the regression, leaving only two parameters for four coefficients. In general, data on observed choices is insufficient to identify the locations of alternatives in models of quadratic utility plus error.

These identification problems led Poole and Rosenthal to develop D-NOMINATE with a quasi-concave utility function. In s dimensions the generalization that nests the Euclidean and directional models is

$$U_{ijy} = u_{ijy} + \varepsilon_{ijy} \tag{10}$$

where

$$u_{ijy} = \exp\left(\beta_1\left[-\sum_{k=1}^{s}\left((x_{ik}-x_o)^2 + (z_{jyk}-x_0)^2\right)\right] + \beta_2\left[2\sum_{k=1}^{s}(x_{ik}-x_o)(z_{jyk}-x_o)\right]\right) \tag{11}$$

We refer to (10) and (11) as the quasi-concave model. This transformation allows for identification in spaces of arbitrary dimension. Poole and Rosenthal (1991) find that at most two dimensions are useful in accounting for congressional roll-call data.[6] In the years covered by this study, over 80% of the observed choices, even on close votes, are accounted for by a one-dimensional model and over 85% by a two-dimensional model.

Results

To preserve comparability with Rabinowitz and Macdonald, we report results for the one-dimensional generalized utility function (equation (7), where s = 1). These results are shown in Table 1. Bootstrap results reported in Poole and Rosenthal (1991) indicate that the β's are very precisely estimated and, as Table 1 indicates, with standard errors less than one-tenth of their magnitudes. The key result is that β_1 is reasonably close to β_2, meaning that quadratic utility is supported and directional utility is rejected. It is important to note that, because of the unusually large data set used in the estimation, traditional significance tests would indicate that $\beta_1 \neq \beta_2$. We stress that the significance of our results lies in the order of magnitude of both β_1 and β_2. Rabinowitz and Macdonald refer to what they call the "model ratio," the ratio of β_2 to β_1. The extent to which this ratio is greater than 1 indicates the divergence from Euclidean to directional voting. Note that this ratio is always less or approximately 1 in both Tables 1 and 2, indicating use of a quadratic utility function.

In fact, β_1 is marginally greater than β_2 in all of the congresses in this study. These results are opposite to what Rabinowitz and Macdonald find in their estimation using the thermometer data and provides further confirmation of our hypothesis.

TABLE 1
One-Dimensional Directional Model of
Voting Behavior in the U.S. House
(standard errors in parentheses)

Congress	Geometric Mean Probability	β_1	β_2	Number of Roll Calls
92d	.722	3.765 (.0104)	2.950 (.0081)	527
93d	.708	3.386 (.0069)	2.969 (.0062)	917
94th	.715	3.792 (.0074)	2.907 (.0056)	1064
95th	.714	4.136 (.0074)	3.078 (.0057)	1217
96th	.725	4.164 (.0143)	3.066 (.0105)	1067
97th	.731	4.733 (.0114)	3.374 (.0082)	679
98th	.748	4.455 (.0103)	3.249 (.0073)	783
99th	.742	3.841 (.0089)	2.989 (.0065)	777

Results from estimation of the quasi-concave utility function (11) in two dimensions are reported in Table 2. Casual observation of the estimates for β_1 and β_2 clearly reveals that the two coefficients are basically identical. Note that in this case β_1 is marginally less than β_2, but the margin is considerably smaller than that in the previous model.

To show the robustness of our results with respect to the choice between quasi-concave and quadratic utility and with respect to the constraints $\beta_1 = \beta_2$ and $\beta_1 \neq \beta_2$, we show correlations of the estimated legislator ideal points. The first two columns of Table 3 show that we get virtually identical one-dimensional estimates of the ideal points whether we use the Rabinowitz and Macdonald model (7) or the quasi-concave model (9). This result holds even if we impose the Poole and Rosenthal (1985, 1991) constraint $\beta_1 = \beta_2$. The last two columns of Table 3 show a similar robustness for two dimensions with regard to the constraint. The estimates are more robust for the first-dimension coordinates (column 3) than for the second-dimension coordinates (column 4). This result is not surprising, since Poole and Rosenthal

TABLE 2
Quasi-Concave Directional Model of Voting Behavior in the U.S. House
(standard errors in parentheses)

Congress	Geometric Mean Probability	β_1	β_2
92d	.715	.8738 (.0030)	1.009 (.0034)
93d	.700	.7703 (.0018)	.8942 (.0022)
94th	.702	.7650 (.0017)	.8723 (.0019)
95th	.699	.7825 (.0016)	.9238 (.0019)
96th	.713	.8977 (.0021)	.9691 (.0022)
97th	.713	.7827 (.0023)	.9059 (.0026)
98th	.731	.8686 (.0025)	.9594 (.0026)
99th	.735	1.008 (.0029)	1.039 (.0027)

(1991) report that the second dimension is less precisely estimated than the first. For the first dimension in the two-dimensional estimates and for all unidimensional estimates, the correlations of the estimated legislator ideal points are all 0.95 or better. These correlations provide strong support for the Poole and Rosenthal quasi-concave model.

Conclusion

The parameter estimates in Tables 1 and 2 clearly indicate that, when the Rabinowitz and Macdonald one-dimensional model and its nonlinear transformation are tested on a body of informed voters, the Rabinowitz and Macdonald directional model can be unambiguously rejected in favor of the traditional Euclidean spatial model. By estimating the same model as Rabinowitz and Macdonald, using congressional votes as data, we have shown that congressional voting

TABLE 3
Correlation Coefficients for Legislator Ideal Points

Congress	Quadratic $\beta_1 \neq \beta_2$ with Quasi-Concave $\beta_1 = \beta_2$ One Dimension	Quadratic $\beta_1 \neq \beta_2$ with Quasi-Concave $\beta_1 \neq \beta_2$ One Dimension	Quasi-Concave $\beta_1 \neq \beta_2$ with Quasi-Concave $\beta_1 = \beta_2$	
			First Dimension	Second Dimension
92d	.98	.95	.93	.88
93d	.99	.99	.99	.90
94th	.99	.99	.99	.95
95th	.99	.99	.99	.95
96th	.99	.99	.99	.96
97th	.98	.99	.98	.84
98th	.99	.99	.99	.99
99th	.98	.99	.98	.90

can be very accurately represented by the Euclidean model. This finding does not mean that the directional model is not appropriate for use in modeling the voting behavior of the uninformed but rather that we can not completely discard the Euclidean model in favor of the directional model. There is another explanation why the Euclidean model is better suited for members of Congress: roll-call votes offer members a choice between two well-defined outcomes—the passage or defeat of the motion. The NES survey questions are considerably more vague and depend on measures of affect. As Rabinowitz and Macdonald's results suggest, there is value in the directional theory and model, but it is our position that informed voters are still better modeled as Euclidean voters.

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NOTES

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1. The early literature is summarized in Enelow and Hinich 1984. See Krehbiel 1988 for specific applications to legislative analysis.

2. This theory would imply that candidates place themselves at the outermost portion of the policy space or that in an unbounded policy space there will be no candidate equilibrium. Rabinowitz and Macdonald 1989 address this issue by assuming that there is a "region of acceptability" that binds a candidate to place himself within a closed subset of the policy space, since the public will be harsher in judging the extremist candidate.

3. The generalized model reflects a conversation between Rabinowitz and Rosenthal following the presentation of an earlier version of the Rabinowitz and Macdonald paper. We appreciate their carrying out the additional analysis that has led to a clear set of comparisons between the quadratic model and the directional model.

4. This tendency of estimated ideal points to cluster near the center does not imply that uninformed voters are all moderates; instead, it implies that they have no consistent voting pattern and consequently their central tendency is an artifact of the estimation procedure.

5. The sample was restricted to roll calls with more than 2.5% of those voting in the minority and to representatives voting at least 25 times.

6. The Poole and Rosenthal model is identical to equation (10) except that $\beta_1=\beta_2$.

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